

Homework #7

Chapter 5: 51, 61, 72, 93
95 (Implicit &
Explicit)

Chapter 6: 1, 12

Problem 5.51

Long Rod 40mm in Diameter (Sapphire)

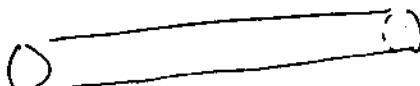
$$k = 22.3 \text{ @ } 550\text{K}$$

$$T_i = 800\text{K}$$

$$T_\infty = 300\text{K}$$

$$h = 1600 \text{ W/m}^2\text{K}$$

$$Bi = \frac{h r_0}{2k} = 0.717 \quad \therefore \text{NOT LUMPED CAPACITANCE}$$



After 35s, the rod is wrapped in insulation.
What is the final equilibrium temperature

$$Bi = \frac{h r_0}{K} = 1.43$$

$$C_p = 1060 \text{ J/kg K} @ 550\text{K}$$

$$\rho = 3970 \text{ kg/m}^3$$

$$Fo = \frac{\alpha t}{r_0^2} = 0.464$$

$$\alpha = 5.3 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Bi^2 Fo = 0.95$$

Figure D.6

$$\frac{Q}{Q_0} = 0.62$$

$$\frac{Q}{Q_0} = \frac{T_{eq} - T_i}{T_\infty - T_e} \Rightarrow T_{eq} = 490\text{K}$$

Problem 5.61 Stainless Steel Ball Bearings

$$T_i = 850^\circ\text{C}$$

$$T_\infty = 40^\circ\text{C}$$

$$h = 1000 \text{ W/m}^2\text{K}$$



$$Bi = \frac{h r_0}{3k} = 0.1554 \quad \begin{matrix} \text{NOT LUMPED} \\ \text{CAPACITANCE} \end{matrix} \quad D = 20\text{mm} \quad r_0 = 0.01\text{m}$$

a) How long does it take for surface to reach 100°C

$$Bi = \frac{h r_0}{k} = 0.466$$

$$\tau_{r_0} = 1 \quad Bi^{-1} = 2.14$$

$$\frac{\Theta}{\Theta_0} = 0.78$$

$$\Theta = T - T_\infty = 60^\circ\text{C}$$

$$\Theta_0 = 76.9$$

$$@ 720\text{K} \quad k = 21.45 \text{ W/mK}$$

$$C_p = 571.75 \text{ J/kgK} \quad \rho = 7900 \text{ kg/m}^3$$

$$\alpha = 4.75 \times 10^{-6} \text{ m}^2/\text{s}$$

$$m = \rho \cdot V$$

$$m = \rho \cdot \frac{4}{3}\pi r_0^3$$

$$m = 0.0331 \text{ kg}$$

$$T_0 = \Theta_0 + T_\infty = 116.9^\circ\text{C}$$

$$\frac{\Theta_0}{\Theta_i} = \frac{76.9}{810} = 0.095$$

$$\text{Figure D.7} \Rightarrow f_0 = 1.9 = \frac{\alpha t}{r_0^2}$$

$$t = 40\text{s}$$

b) If 1000 Balls are quench per hour find rate of heat removed from oil bath

$$Q_0 = m C_p [T_i - T_\infty]$$

$$Q_0 = 1.53 \times 10^4 \text{ J}$$

$$Bi^2 f_0 = 0.412$$

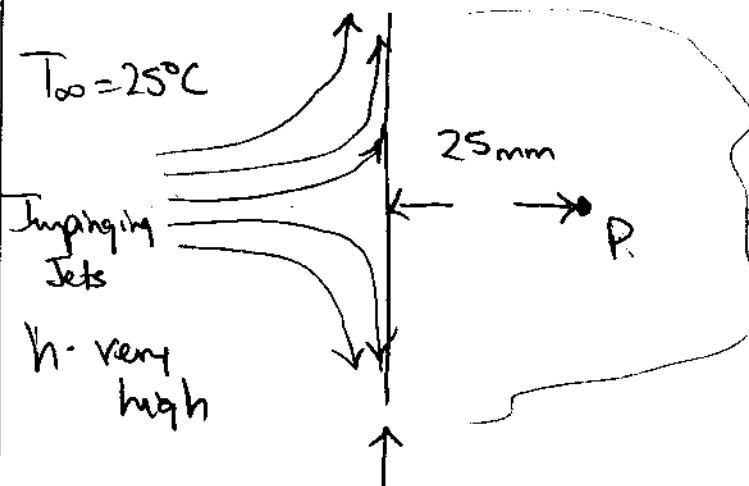
$$\frac{Q}{Q_0} = 0.9$$

$$q = Q_0 \cdot \frac{Q}{Q_0} \cdot \frac{1000}{\text{hour}} \cdot \frac{\text{hour}}{3600\text{s}}$$

$$[q = 38.3 \text{ kW}]$$

Prob 5.72

Thick Steel Slab



$$k = 50 \text{ W/mK}$$
$$\rho = 7800 \text{ kg/m}^3$$
$$c_p = 480 \text{ J/kg K}$$

$$T_i = 300^{\circ}\text{C}$$

$$\alpha = 1.33 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{assume } T_s = \text{Const} = 25^{\circ}\text{C}$$

How long will it take for the temperature at P to reach 50°C

$$\frac{T(x,t) - T_s}{T_i - T_s} = \operatorname{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right)$$

$$\operatorname{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right) = 0.091$$

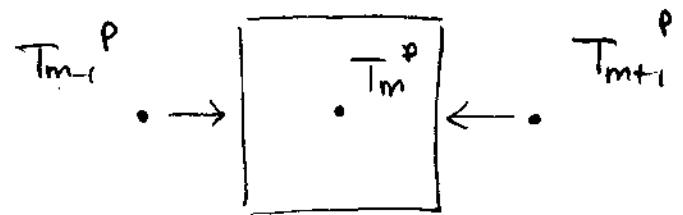
Table B.2

$$\frac{x}{2\sqrt{\alpha t}} = 0.0807$$

$$t = \frac{1}{\alpha} \left(\frac{x}{2 \cdot 0.0807} \right)^2$$

$$t = 1802 \text{ seconds}$$

Problem 5.93



$$\bar{T}_{m-1}^P = 100^\circ\text{C} \quad \bar{T}_m^P = 50^\circ\text{C} \quad \bar{T}_{m+1}^P = 100^\circ\text{C}$$

Show that for values of $F_0 > 1/2$ the explicit method violates the second law of thermodynamics

$$f_0 \Delta x \left[\frac{\bar{T}_m^{P+1} - \bar{T}_m^P}{\Delta t} \right] = K \left[\frac{\bar{T}_{m-1}^P - \bar{T}_m^P}{\Delta x} \right] + K \left[\frac{\bar{T}_{m+1}^P - \bar{T}_m^P}{\Delta x} \right]$$

$$\bar{T}_m^{P+1} = F_0 \bar{T}_{m-1}^P + (1 - 2F_0) \bar{T}_m^P + F_0 \bar{T}_{m+1}^P$$

$$F_0 = \frac{1}{2}$$

$$\bar{T}_m^{P+1} = \frac{1}{2} \bar{T}_{m-1}^P + \frac{1}{2} \bar{T}_{m+1}^P$$

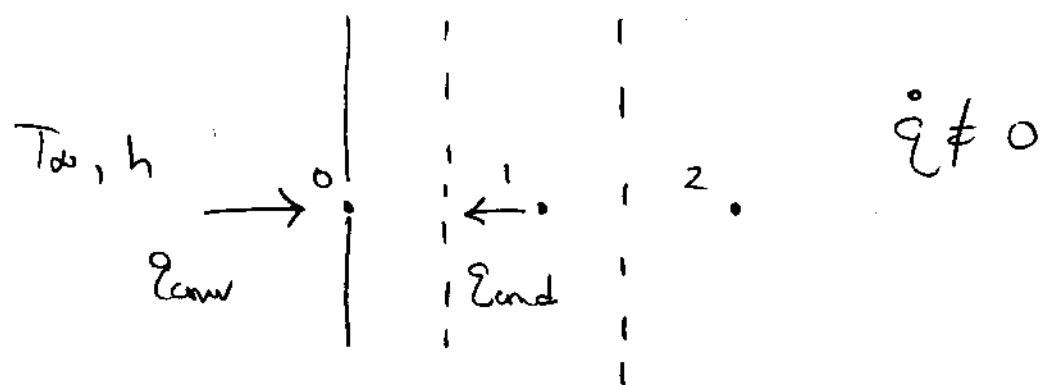
$$\bar{T}_m^{P+1} = 50^\circ\text{C} + 50^\circ\text{C} + 50^\circ\text{C} = 100^\circ\text{C}$$

$$F_0 = 1$$

$$\bar{T}_m^{P+1} = \bar{T}_{m-1}^P - \bar{T}_m^P + \bar{T}_{m+1}^P$$

$$\bar{T}_m^{P+1} = 250^\circ\text{C} \quad \leftarrow \text{Violates 2nd Law}$$

Problem 5.95 1D Slab



$$\rho C_p \frac{\Delta x}{2} \alpha \left[\frac{T_m^{P+1} - T_m^P}{\Delta t} \right] = h \alpha [T_{\infty} - T_m^P] \\ + K \alpha \left[\frac{T_m^{P+1} - T_m^P}{\Delta x} \right] \\ + \dot{q} \frac{\Delta x}{2} \alpha$$

$$T_m^{P+1} - T_m^P = \frac{2h\Delta t}{\rho C_p \Delta x} [T_{\infty} - T_m^P]$$

$$+ 2K \frac{\Delta t}{\rho C_p \Delta x^2} \left[\frac{T_m^{P+1} - T_m^P}{\Delta x} \right]$$

$$+ \dot{q} \frac{\Delta t}{\rho C_p} \quad F_o = \frac{K}{\rho C_p} \frac{\Delta t}{\Delta x^2} \\ B_i = \frac{h \Delta x}{K}$$

$$T_m^{P+1} = 2F_o T_m^{P+1} + (1 - 2F_o - 2F_o B_i) T_m^P \\ + 2F_o B_i T_{\infty} + \dot{q} \frac{\Delta t}{\rho C_p} \quad \text{EXPLICIT}$$

IMPLICIT

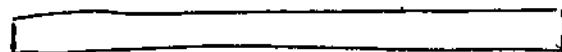
$$q \frac{C_p \Delta x}{2} A \left[\frac{T_m^{p+1} - T_m^p}{\Delta t} \right] = hA \left[T_\infty - T_m^{p+1} \right] \\ + KA \left[\frac{T_{m+1}^{p+1} - T_m^{p+1}}{\Delta t} \right] + \dot{q} \frac{\Delta x}{2} A$$

$$- 2F_o T_{m+1}^{p+1} + (1 + 2F_o + 2F_o Bi) T_m^{p+1}$$

$$= 2F_o Bi T_\infty + \dot{q} \frac{\Delta t}{C_p} + T_m^p$$

Problem 6.1 Laminar Flow Over Flat Plate

$$h_x \propto x^{-1/2} = C x^{-1/2}$$



$\rightarrow x$

What is the ratio of \bar{h}/h_x

$$\bar{h} = \frac{1}{L} \int_0^L C x^{-1/2} dx$$

$$\bar{h} = \frac{1}{L} \cdot C \left[2x^{1/2} \right]_0^L = \frac{2C}{L^{1/2}}$$

$$h_x \Big|_{x=L} = \frac{C}{L^{1/2}}$$

$$\frac{\bar{h}}{h_x \Big|_{x=L}} = \frac{2C/L^{1/2}}{C/L^{1/2}} = 2$$

Problem 6.12 Temperature Distribution

$$\frac{T - T_s}{T_\infty - T_s} = 1 - \exp \left(-Pr \frac{u_\infty y}{V} \right)$$

$$Pr = 0.7$$

$$\frac{u_\infty}{V} = 5000 \text{ 1/m}$$

$$T_\infty = 400\text{K} \quad T_s = 300\text{K}$$

What is the surface heat flux

$$T = (T_\infty - T_s) \left[1 - \exp \left(-Pr \frac{u_\infty}{V} y \right) \right] + T_s$$

$$\frac{dT}{dy} = (T_\infty - T_s) \cdot Pr \frac{u_\infty}{V} \exp \left(Pr \frac{u_\infty}{V} y \right)$$

$$\left. \frac{dT}{dy} \right|_{y=0} = (T_\infty - T_s) \cdot Pr \frac{u_\infty}{V} = 3.5 \times 10^5 \frac{\text{K}}{\text{m}}$$

$$k_f = 0.0263 \frac{\text{W}}{\text{mK}} @ 300\text{K}$$

$$q'' = -k_f \left. \frac{dT}{dy} \right|_{y=0} = \boxed{-9205 \frac{\text{W}}{\text{m}^2}}$$

$$q'' = h [T_s - T_\infty]$$

$$\boxed{h = 92 \frac{\text{W}}{\text{m}^2 \text{K}}}$$